What do mathematicians do?

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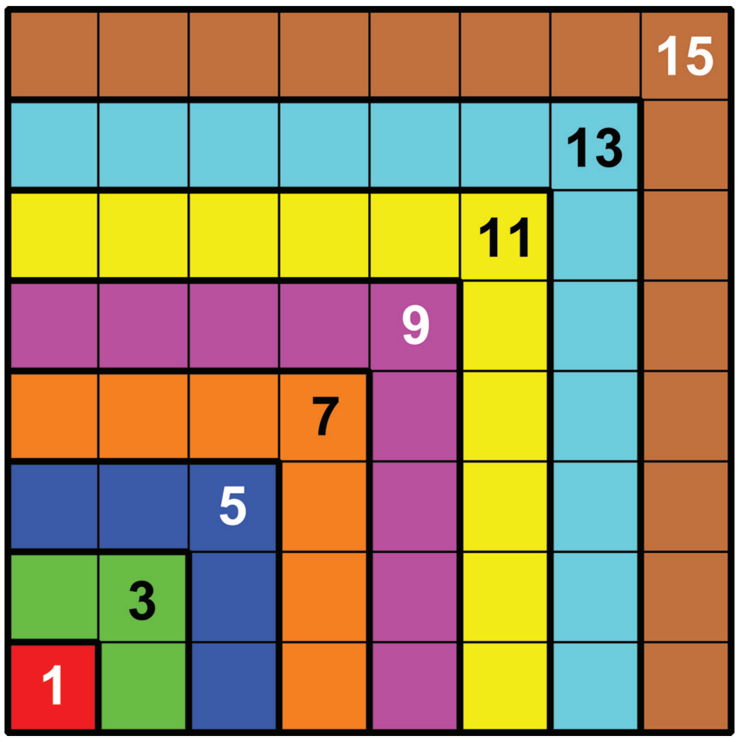
"A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas."

This is what G. H. Hardy, in his 1941 book *A Mathematician's Apology*, wrote to describe what a mathematician does. What kind of patterns and ideas is Hardy talking about?

Mathematicians find patterns in imaginary objects. Mathematicians do not care much that their work has little to do with real life. Scientists from various fields usually model their real-life scenarios around certain mathematical frameworks, but mathematicians are happy to discover patterns in objects that may have no real-world importance. Like music, poetry, dance and various other art forms, mathematics is a form of art.

For example, I may be interested in obtaining the sum of the first one thousand positive odd numbers. (An odd number is a number that is not evenly divisible by two.) Why I want to do this does not matter. Like a music composer who writes songs because he or she is in the mood for doing so, I want to find this sum because I am curious – I have no external application motivating me.



A geometric proof that the sum of the first *n* odd numbers is the square of *n*.

How on earth am I going to obtain this sum? Do I have to really sum up one thousand numbers? Is there a shortcut? Is there some pattern I can use? What if I had only five odd numbers? In that case, 1+3+5+7+9 is easy: 25. What if I had eight? Well, these would sum up to 25+11+13+15, or 64.

Wait a minute. The sum of the first five odd numbers is 25... which is five times five! And the sum of the first eight odd numbers is eight times eight! There may be a pattern here! Can I prove that this pattern always works? Why is this happening?

Maybe I can sum up my numbers in a different order, or draw some diagram, to see this hypothetical pattern emerge. Perhaps I draw the diagram shown in the figure to help me prove that the sum of the first *n* odd numbers is indeed the square of *n*. Armed with this proof, I can then confidently say that the sum of the first one thousand odd numbers is one million.

This process of thought, discovery and proof of speculative patterns, is the essence of the mathematician's work.

Ernst Kummer. Image source: https://rjlipton.files.wordpress.com/2016/01/ernst-eduard-kummer2.jpg

MYTH DEBUNKED

Are all mathematicians good at calculation?

People often think that being good at quickly adding or multiplying numbers is a sign of being a mathematical genius. The truth is that mathematics has little to do with numbers, and more to do with spotting patterns and proving results. Indeed, evaluating complicated calculations quickly has practically no mathematical value whatsoever, so much so that nowadays we let calculators and computers perform this tedious work for us.

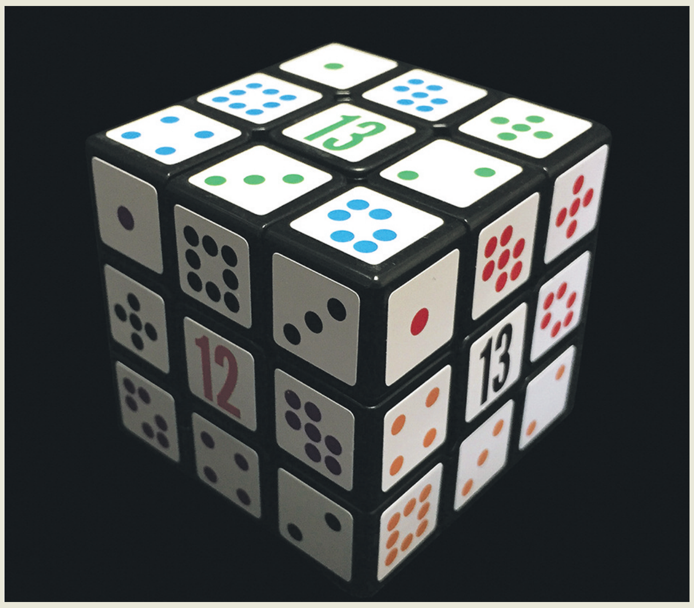
Nevertheless, it is true that mathematicians are good at performing quick number calculations. However, one mathematician who is an exception to this rule is Ernst Kummer, a German mathematician who introduced ideal numbers – a concept later extended by Richard Dedekind.

By doing so, Kummer managed to partially prove a taxing claim made by Pierre de Fermat, nowadays known as Fermat's Last Theorem. This result was only successfully proved completely in 1994 by Sir Andrew Wiles, for which he received the Abel prize last year.

However, Kummer's arithmetic was very poor. Once, in class, he needed the product 7x9. He could not remember it, so he asked the class to help him. One mischievous student told him that the answer is 61. Another student interjected "no, the correct answer is 69". Eventually, he figured out that the correct answer is 63 by using the following method: 61 is a prime number, 65 is divisible by 5, 67 is a prime number, 69 is too large, so it must be 63.

This shows that even the best of mathematicians may not be as good with multiplying numbers as you think!

PHOTO OF THE WEEK



If you enjoy solving Rubik's cube, then you could attempt to solve the Grime cube. The Grime cube was invented last year by Dr James Grime, a mathematician and public speaker promoting mathematics and residing in the University of Cambridge. His cube can be solved in two ways. The first way is to align the cube so that its faces are the same colour, akin to the usual Rubik's cube. The second way of solving the cube is to arrange each face so that it contains a correct perimeter magic square. A perimeter magic square is a three-by-three grid filled with different numbers such that the sum of the numbers in the top row, bottom row, left column and right columns is equal to the number in the centre. The challenge is to go back and forth from one solved state to the other.

PHOTO: https://cdn.shopify.com/s/files/1/0222/9748/products/grime-cubes01.jpg

SOUND BITES

- *From the international scene*: The Kepler conjecture, informally, states that the densest way to pack spheres is to either use cubic (where the spheres form a cube-like shape) or hexagonal (forming a hexagonal shape) close packing. This conjecture was made in 1611 by Johannes Kepler (1571-1630). Unfortunately, this assertion was found to be very difficult to prove. In 1992, Thomas Hales and his student Samuel Ferguson from the University of Pittsburgh, Pennsylvania embarked on a proof of this result. After solving thousands of sub-cases with the aid of computers, Hales successfully submitted a paper in 1998 claiming that the conjecture is true. However, the proof was so long that the referees only managed to verify its non-computing part by 2003, and they could not verify the entire proof, although they did publish the paper in 2005: *Thomas C. Hales, A proof of the Kepler conjecture, Annals of Mathematics 162 (2005), 1065-1185.* Saddened by the fact that the conjecture was not completely set in stone, Hales embarked on another project called *Flyspeck*, which attempted to verify the computer part of the proof using proof checking software. In 2014, he announced that the verification was complete, and only recently, in June 2017, this part of the proof was finally published: *Thomas Hales et al, A formal proof of the Kepler conjecture, Forum of Mathematics, Pi (2017).*

- *From the local scene*: In mathematical chemistry, the total pi-electron energy of a molecule can be obtained by a formula originally described in *C. A. Coulson, On the calculation of the energy in unsaturated hydrocarbon molecules, Proc. Cambridge Phil. Soc. 36 (1940), 201-203.* In the recent paper *Alexander Farrugia, Coulson-type Integral Formulae for the Energy Changes of Graph Perturbations, MATCH Commun. Math. Comput. Chem. 77 (2017), 575-588*, formulas of a similar nature to those of Coulson were presented. They provide the change in energy of a molecule after slight changes to the molecule are made. Although the original Coulson formula was discovered almost 80 years ago, it is only now that mathematical chemists can precisely predict this change in molecular energy by a mathematical formula.

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DID YOU KNOW?

- Godfrey H. Hardy (1877-1947), the author of *A Mathematician's Apology*, is portrayed by actor Jeremy Irons in the 2015 movie *The Man Who Knew Infinity*. This film showcases the life of the brilliant Indian mathematician Srinivasa Ramanujan (1887-1920). Mathematicians John Littlewood (1885-1977) and Bertrand Russell (1872-1970) also feature in this film.
- The exclamation mark (!) in mathematics is called the factorial. It is used to denote the multiplication of all the numbers from one to a given number. For example, 4! stands for 1x2x3x4, or 24.
- If a number is made up of a block of digits that repeats several times, then it is always divisible by the number represented by that block of digits. For example, the number 777 is divisible by 7, the number 656565 is divisible by 65, and the number 999999 is divisible by 9, by 99 and by 999.
- If any 1-digit number is divided by 9, then a decimal that repeats that digit forever is obtained. For example, 4/9=0.4444... Similarly, any two-digit number divided by 99 is equal to a decimal that repeats those two digits forever; for example, 82/99=0.828282... This can be extended to any number of digits.

For more trivia see: www.um.edu.mt/think

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